Stats3Y03/3J04 Test 1 (Version 1)

Instructor: Mu He **Time: 7:00 - 8:00 P.M. May 14th, 2018**

First Name:____

Last Name:____

Student ID:____

There are total 16 multiple choice questions for this test. Each question carries equal marks. All questions must be answered on the COMPUTER CARD with an HB PENCIL. You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio FX-991 MS or MS Plus is allowed.

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1. Suppose that a a 6 character passcode consists of 6 digits from 0 to 9, with no repeated digits. If such a passcode is randomly selected, find the probability that it contains the digits '7' and '8' or it ends with a '5'.

(a) 0.4611 (b) 0.4056 (c) 0.4333 (d) 0.4828 (e) 0.4623

2. Suppose that a factory has a group of 105 eletrical components, containing 20% default, if 15 pieces are randomly selected (**with replacement**). Let X denote the number of default items among the chosen ones. What is the variance of X?

(a) 1.94 (b) 2.40 (c) 16.80 (d) 2.08 (e) 2.35

3. Suppose that we have a poker set (52 cards: 13 hearts, 13 diamonds, 13 spade and 13 club). We randomly draw 4 cards (**with replacement**) from the deck and let X denote the number of diamonds we have among the 4 chosen cards. Find the probability that $P(1 \leq X \leq 3)$.

(a) 0.3160625 (b) 0.6935414 (c) 0.6796875 (d) 0.6839375 (e) 0.6328125

4. An batch of 50 eggs containing 4 broken ones, if a random selection is performed (**with replacement**) untill we get 3 broken ones. What is the probability that we select 5 times when we get 3 broken eggs.

(a) 0.00271 (b) 0.00293 (c) 0.00282 (d)0.00260 (e) 0.00254

5. On average, the shoppers across McMaster Univerisity have 2 customers per hour and assuming that for the next hour the number of customers denoted by X, follows a Poisson Distribution. Find the probability that at least two customers are there for the next hour.

(a) 0.2707 (b) 0.4060 (c) 0.5413 (d) 0.7293 (e) 0.5940

6. Shafts are classied in terms of the machine tool that was used for manufacturing the shaft and conformance to surface finish and roundness.

Tool1		Roundness Conforms	
		yes	no
Surface Finish Conforms	yes	200	1
	no	4	2

Tool2		Roundness Conforms	
		yes	no
Surface Finish Conforms	yes	145	4
	no	8	6

A shaft is selected at random. Let E_1 be the event that Tool 1 was used, Let E_2 be the event that it conforms to roundness specifications, and let E_3 be the event that it conforms to surface finish specifications. Find $P[E_1 \cup (E_2' \cap E_3)]$.

(a) 0.5811 (b) 0.8327 (c) 0.5703 (d) 0.5228 (e) 0.6415

7. Suppose there are total 8% people in McMaster University are left-handed, we randomly ask student in campus and let X be the number of students we have asked when we get the first left-handed person. Find $E(X^2)$.

(a) 143.75 (b) 12.5 (c) 156.25 (d) 200 (e)300

8. Let A_1, A_2, A_3 be three different kinds of defects that we can have, where $P(A_1) = 0.28$, $P(A_2) = 0.39$, $P(A_3) = 0.35$, $P(A_1 \cup A_2) = 0.59$, $P(A_1 \cup A_3) = 0.6$, $P(A_2 \cup A_3) = 0.63$, $P(A_1 \cap A_2 \cap A_3) = 0.01$, find the probability that the system has exactly 2 of 3 types of defects.

(a) 0.18 (b) 0.19 (c) 0.20 (d) 0.21 (e) 0.17

9. Suppose that the diameter of a hole (in millimeters) drilled in a sheet metal component has probability density function:

$$f(x) = \frac{ce^{-(x-6)}}{1+e^{-(x-6)}}, \qquad x \geq 6$$

If we drill 20 such holes, find the variance of the number of the holes that have a diameter greater than 8 millimeters.

(a) 8.95 (b) 3.66 (c) 2.99 (d) 2.22 (e) 4.49

10. Suppose the hospital have 10 patients waiting for surgeries, 3 of them are knee surgeries (Identical), 4 of them of hip surgeries (Identical), 3 of them are others (Different from each other). However, due to the limitation of the doctors, the hospital can only perform 2 out of 3 for the others, which means that there will be 3 knee surgeries, 4 hip surgeries and just 2 others surgeries in total on schdule. How many ways to arrange the 9 surgeries. (Assuming that knee surgeries are identical and hip surgeries are identical, others are different with each other, all schedules are equally likely)

(a)7560 (b) 2520 (c) 25200 (d) 10080 (e) 15120

11. Assuming that left-handed and gender are independent events, suppose the probability of left-haned people are 10% and there are 55% female for a certain department. What is the probability if one is randomly chosen from the department that the person is either left-handed or female.

(a) 0.65 (b) 0.6445 (c) 0.6 (d) 0.55 (e) 0.595

12. Suppose that a password is exactly six characters and each character is 1 of the 26 lowercase letters (a-z) or 10 integers (0-9). How many passwords contain exactly 1 interger?

(a) 657800 (b) 3946800 (c)154355993535744000 (d) 712882560 (e)118813760

13. Let X denote the vibratory stress (psi) on a wind turvine blade at a particular wind speed in a wind tunnel. Suppose that X has p.d.f.:

$$f(x) = \frac{x}{100} e^{-x^2/200}, \qquad x > 0$$

Then, 80% of time, the vibratory stress is less than what value?

(a) 6.68 (b) 21.56 (c) 17.94 (d) 5.12 (e) 8.92

14. A machine produces corks (for use in wine bottles) with diameters that are normally distributed with mean 3 cm and standard deviation 0.1 cm. What percentage of such corks are between 2.85 cm and 3.1 cm ?

(a) 0.3523 (b) 0.0918 (c) 0.7745 (d) 0.8416 (e) 0.5781

15. Knowing C.D.F

$$F(x) = \begin{cases} 0 & x < 3 \\ \frac{c}{27}(x^2 - 9) & 3 \le x < 6 \\ 1 & x \ge 6 \end{cases}$$

what is the variance for X?

(a) $\frac{14}{3}$ (b) $\frac{13}{18}$ (c)$\frac{797}{18}$ (d) 1 (e) $\frac{45}{2}$

16. The answer for this question is (a), if you could bubble both this question's answer and the version number right, you could get the mark for this question.

Answer: 1(b) 2(b) 3(c) 4(d) 5(e) 6(c) 7(e) 8(b) 9(c) 10(a) 11(e) 12(d) 13(c) 14(c) 15(b) 16(a)

Test 1 Formula Sheet

Probability Rule:

Probability of Union (Two): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability of Union (Three): $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Independence: $P(A \cap B) = P(A) \times P(B)$ or $P(B|A) = P(B)$

Total Probability Rule: Suppose $E_1, E_2, ..., E_k$ are k exhaustive and mutually exclusive events, then $P(B) = P(B \cap E_1) + P(B \cap E_2) + ... + P(B \cap E_k) = P(B|E_1)P(E_1) + ... + P(B|E_k)P(E_k)$

Discrete R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \sum_x xf(x)$

Variance: $\mathbb{V}(X) = \sigma^2 = \sum_x (x-\mu)^2 f(x) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \sum_{y:y<x} f(x)$

Continuous R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$

Common Distributions:

Binomial Distribution (n,p):

$f(x) = \binom{n}{k} p^x (1-p)^{n-x}$, $x = 0, 1, 2, ..., n$

$\mathbb{E}(X) = np$, $V(X) = np(1-p)$

Hypergeometric Distribution (n,K,N):

$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$, $x = \max\{0, n+K-N\}$ to $\min\{K, n\}$

$\mathbb{E}(X) = np$, $V(X) = np(1-p)\frac{N-n}{N-1}$, where $p = \frac{K}{N}$

Geometric Distribution (p):

$f(x) = p(1-p)^{x-1}$, $x = 0, 1, 2, ...$

$\mathbb{E}(X) = \frac{1}{p}$, $\mathbb{V}(X) = \frac{1-p}{p^2}$

Negative Binomial Distribution (r,p):

$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$, $x = 0, 1, 2, ...$

$\mathbb{E}(X) = \frac{r}{p}$, $\mathbb{V}(X) = \frac{r(1-p)}{p^2}$

Poisson Distribution (λ):

$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2, ...$

$\mathbb{E}(X) = \lambda$, $\mathbb{V}(X) = \lambda$

Normal Distribution (μ, σ^2):

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

$\mathbb{E}(X) = \mu$, $\mathbb{V}(X) = \sigma^2$

Exponential Distribution (λ):

$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$

$\mathbb{E}(X) = \frac{1}{\lambda}$, $\mathbb{V}(X) = \frac{1}{\lambda^2}$

Standard Normal Probabilities

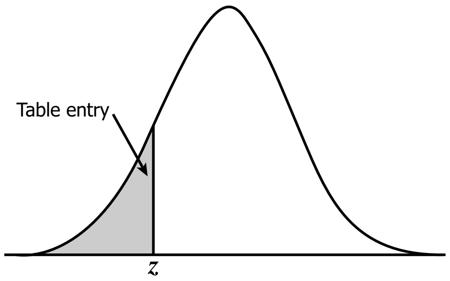

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

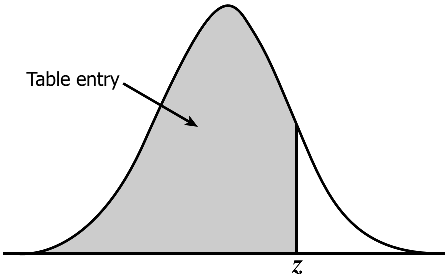

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998